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# Spectrum of the parametric down converted radiation calculated in the Wigner function formalism

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**Abstract.** We continue the study of parametric down conversion within the framework of the Wigner representation, by using a Maxwellian approach developed in a recent paper [A. Casado *et al.*, Eur. Phys. J. D **11**, 465 (2000)]. This gives a mechanism, inside the crystal, for the production of the down-converted radiation. We obtain the electric field to second order in the coupling constant by using the Green's function method, and compare our treatment with the standard Hamiltonian approach. The spectrum of the down-converted radiation is calculated as a function of the parameters of the nonlinear crystal (in particular the length) and the radius of the pumping beam.

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## 1 Introduction

For many years the state of the radiated field corresponding to quantum parametric down-conversion (PDC from now on) has been studied. A *single* monochromatic laser converts into pairs of highly correlated photons fulfilling the frequency matching conditions [1,2]. PDC is usually considered as a typical quantum phenomena, not only because there is no solution of the classical Maxwell equations which represents the so called "spontaneous splitting" of the laser, but also because there is a very short correlation time between the conjugate beams [3,4] and a high visibility of interference patterns in joint detection experiments [5]. These properties have been used in order to test Bell's inequalities [6] and to show other nonclassical aspects of the down-converted light [7].

The theory of PDC in the Wigner function formalism of quantum optics was treated in an earlier series of papers by using a standard Hamiltonian approach [8–11]. We described how the radiated field is produced *via* the coupling between the laser beam and the zeropoint radiation inside the crystal. We also studied the process of light detection, stressing the fact that all detectors integrate the light intensity over a large time window [11]. We should recall that the Hamiltonian formalism was originally developed, during the 1960's, in parallel [13] with a treatment, rather similar to ours, which also took account of the zeropoint field. Indeed the name by which PDC was known during that period was *spontaneous parametric fluorescence*, and such ways of describing the interaction persist up to the present day.

More recently we have developed the theory of PDC by starting from the Maxwell equations inside the crystal, in place of the usual Hamiltonian standard model [12]. We showed that the production and propagation of PDC light is entirely equivalent to classical electromagnetic field theory, provided that we consider the zeropoint field entering the crystal along with the laser beam. An explicit expression for the first order electric field amplitude in the far field approximation was obtained by using the Green's function method, in order to calculate the cross-correlation of photon counts.

Here we shall use the same formulation in order to study the spectral properties of the PDC radiation. In Section 2 we summarize the main results of the Wigner representation of second-order nonlinear optical phenomena in the Maxwellian approach [12], and in Section 3 we calculate the electric field to second order in the coupling parameter. In addition to the part of the field that corresponds to the splitting of the laser (PDC), there appear some new contributions coming from the up and down conversion of the zeropoint field. In Section 4 we analyze the relation between the different contributions of the field to the autocorrelation, and a comparison is made with the results in the Hamiltonian approach. Finally, in Section 5 we obtain the PDC spectrum, *i.e.* the intensity of the down-converted radiation, at each point, as a function of the frequency, and we consider the long and short crystal approximations.

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## 2 Wigner representation of PDC

The study of PDC in the Wigner representation within a Maxwellian approach starts with the evolution equation for the electric field operator in the Heisenberg picture. By considering a second-order nonlinear isotropic medium, the corresponding equation is [12]

$$\nabla^2 \hat{E} - \frac{1}{c_\mathrm{f}^2}\frac{\partial^2 \hat{E}}{\partial t^2} - \frac{\partial^2}{\partial t^2}\left[\int_{-\infty}^{t} \chi(t-t')\hat{E}(t')\mathrm{d}t'\right] = \beta \frac{\partial^2 \hat{E}^2}{\partial t^2}, \tag{1}$$

$c_\mathrm{f}$ being the speed of light in free space. $\chi$ is the linear susceptibility of the medium, and $\beta$ is a coupling constant which is defined by

$$\beta \equiv 2\mu_0 d, \tag{2}$$

where $d$ is the bilinear susceptibility and $\mu_0$ the magnetic permeability of free space.

Equation (1) is an inhomogeneous wave equation in which the source of radiation is a quadratic function of $\hat{E}$. In order to solve it an adiabatic switch on of the interaction is considered by substituting $\lambda(t)\beta$ for $\beta$, $\lambda(t)$ being a slowly varying function of time, so that $\lambda(t) = 0$ at $t \to -\infty$, and $\lambda(t) = 1$ at $t \geq 0$. The state of the radiation at $t \to -\infty$ is that corresponding to a laser, *i.e.* a coherent state $|\phi\rangle$, fulfilling

$$\hat{E}^{(+)}|\phi\rangle = E_\mathrm{laser}^{(+)}|\phi\rangle, \tag{3}$$

$\hat{E}^{(+)}$ being the part of the electric field operator that only contains destruction operators:

$$\hat{E}^{(+)} = \mathrm{i}\sum_{\mathbf{k}} \left(\frac{\hbar\omega_\mathbf{k}}{\epsilon_0 L_0^3}\right)^{\frac{1}{2}} \hat{a}_\mathbf{k} \mathrm{e}^{-\mathrm{i}\omega_\mathbf{k} t + \mathrm{i}\mathbf{k}\cdot\mathbf{r}}, \tag{4}$$

$L_0^3$ being the normalization volume, and $\omega_\mathbf{k} = |\mathbf{k}|c_\mathrm{f}$.

It is possible to use the vacuum field as the initial state if we perform the following change of variables:

$$\hat{E}^{(+)} = \hat{E}'^{(+)} + E_\mathrm{laser}^{(+)}. \tag{5}$$

By substituting (5) into equation (1), we have

$$\nabla^2(\hat{E}' + E_\mathrm{laser}) - \frac{1}{c_\mathrm{f}^2}\frac{\partial^2(\hat{E}' + E_\mathrm{laser})}{\partial t^2} - \frac{\partial^2}{\partial t^2}\left[\int_{-\infty}^{t} \chi(t-t')[\hat{E}'(t') + E_\mathrm{laser}(t')]\mathrm{d}t'\right] = \lambda(t)\beta\frac{\partial^2(\hat{E}' + E_\mathrm{laser})^2}{\partial t^2} \cdot \tag{6}$$

Let us restrict our attention to equation (6) for $t \geq 0$. If there were no laser beam incoming to the nonlinear medium, *i.e.* if we made $E_\mathrm{laser} = 0$ in equation (6), then this equation would represent the evolution of the vacuum due to the presence of the crystal, and would give rise to just a modified vacuum. If we take into account that the laser is very intense, it seems reasonable to discard the term $\beta\partial^2\hat{E}'^2/\partial t^2$ from (6), because its contribution to the radiated field is very small compared with the others. Hence equation (6) reduces to the following equation, which is linear in the field operators:

$$\nabla^2(\hat{E}' + E_\mathrm{laser}) - \frac{1}{c_\mathrm{f}^2}\frac{\partial^2(\hat{E}' + E_\mathrm{laser})}{\partial t^2} - \frac{\partial^2}{\partial t^2}\left[\int_{-\infty}^{t} \chi(t-t')[\hat{E}'(t') + E_\mathrm{laser}(t')]\mathrm{d}t'\right] = \beta\frac{\partial^2(2E_\mathrm{laser}\hat{E}' + E_\mathrm{laser}^2)}{\partial t^2} \cdot \tag{7}$$

Let us now pass to the Wigner representation. As is well-known, the evolution equations of the Wigner field amplitudes are the same as the Heisenberg equations of motion of the quantum field amplitudes, whenever these are linear. Then, in order to go to the Wigner representation we simply remove the hats in equation (7) (we shall remove also the prime in order to simplify the notation), so that

$$\nabla^2(E + E_\mathrm{laser}) - \frac{1}{c_\mathrm{f}^2}\frac{\partial^2(E + E_\mathrm{laser})}{\partial t^2} - \frac{\partial^2}{\partial t^2}\left[\int_{-\infty}^{t} \chi(t-t')[E(t') + E_\mathrm{laser}(t')]\mathrm{d}t'\right] = \beta\frac{\partial^2(2E_\mathrm{laser}E + E_\mathrm{laser}^2)}{\partial t^2} \cdot \tag{8}$$

Because we are in the Heisenberg picture the state does not change with time and we shall use the Wigner function corresponding to the initial state, *i.e.* the Wigner function of the vacuum state

$$W_\mathrm{vacuum}(\{\alpha_\mathbf{k}\}, \{\alpha_\mathbf{k}^*\}) = \prod_\mathbf{k} \frac{2}{\pi}\mathrm{e}^{-2\alpha_\mathbf{k}\alpha_\mathbf{k}^*}, \tag{9}$$

$\alpha_\mathbf{k}$ being the complex amplitude corresponding to the mode $\mathbf{k}$ of the zeropoint radiation

$$E_\mathrm{ZP} = E_\mathrm{ZP}^{(+)} + E_\mathrm{ZP}^{(-)},$$

where

$$E_\mathrm{ZP}^{(+)} = \mathrm{i}\sum_\mathbf{k}\left(\frac{\hbar\omega_\mathbf{k}}{\epsilon_0 L_0^3}\right)^{\frac{1}{2}}\alpha_\mathbf{k}\mathrm{e}^{-\mathrm{i}\omega_\mathbf{k}t + \mathrm{i}\mathbf{k}\cdot\mathbf{r}}; \quad E_\mathrm{ZP}^{(-)} = [E_\mathrm{ZP}^{(+)}]^*. \tag{10}$$

$E_\mathrm{ZP}^{(+)}$ is the positive frequency part of the vacuum field. On the other hand, from (9) it follows trivially that

$$\langle\alpha_\mathbf{k}\rangle = 0; \quad \langle\alpha_\mathbf{k}\alpha_{\mathbf{k}'}\rangle = 0; \quad \langle\alpha_\mathbf{k}\alpha_{\mathbf{k}'}^*\rangle = \frac{1}{2}\delta_{\mathbf{k},\mathbf{k}'}. \tag{11}$$

## 3 Second-order perturbation theory for the calculation of the radiated field

The electric field can be expressed as an expansion in powers of the small parameter $\beta$ [9]:

$$E(\mathbf{r},t) = E_0(\mathbf{r},t) + \beta E_1(\mathbf{r},t) + \beta^2 E_2(\mathbf{r},t) + ... \quad (12)$$

Now a set of coupled equations is obtained by substituting (12) into (8):

$$\nabla^2(E_0 + E_{\text{laser}}) - \frac{1}{c_{\text{f}}^2}\frac{\partial^2(E_0 + E_{\text{laser}})}{\partial t^2} - \frac{\partial^2}{\partial t^2}\left[\int_{-\infty}^{t} \chi(t-t')[E_0(t') + E_{\text{laser}}(t')]\text{d}t'\right] = 0, \quad (13)$$

$$\nabla^2 E_1 - \frac{1}{c_{\text{f}}^2}\frac{\partial^2 E_1}{\partial t^2} - \frac{\partial^2}{\partial t^2}\left[\int_{-\infty}^{t} \chi(t-t')E_1(t')\text{d}t'\right] = \frac{\partial^2(2E_0 E_{\text{laser}} + E_{\text{laser}}^2)}{\partial t^2}, \quad (14)$$

$$\nabla^2 E_2 - \frac{1}{c_{\text{f}}^2}\frac{\partial^2 E_2}{\partial t^2} - \frac{\partial^2}{\partial t^2}\left[\int_{-\infty}^{t} \chi(t-t')E_2(t')\text{d}t'\right] = 2\frac{\partial^2(E_1 E_{\text{laser}})}{\partial t^2}, \quad (15)$$

and so on. To zeroth order in $\beta$, $E_0$ is given by equation (10), *i.e.* $E_0 \equiv E_{\text{ZP}}$. On the other hand, from now on we shall consider $E_{\text{laser}}$ as a quasimonochromatic beam of frequency $\omega_0$, wave vector $\mathbf{k}_0$, and radius $R$:

$$E_{\text{laser}} = E_{\text{laser}}^{(+)} + E_{\text{laser}}^{(-)},$$

where

$$E_{\text{laser}}^{(+)}(\mathbf{r},t) = V(\mathbf{r})\,\text{e}^{-\text{i}\omega_0 t + \text{i}\mathbf{k}_0\cdot\mathbf{r}};$$
$$V(\mathbf{r}) = V_0\,\text{e}^{-\frac{x^2+y^2}{2R^2}};\ \mathbf{k}_0 = \frac{\omega_0}{c_0}\mathbf{u}_z, \quad (16)$$

$c_0$ being the velocity of light corresponding to the frequency $\omega_0$. We are taking a coordinate system O$XYZ$, O being the center of the crystal and $\mathbf{u}_z$ an unitary vector in the direction of the pumping. Strictly speaking (16) is a solution of the homogeneous wave equation only in the limit case $R \to \infty$ [1], but we follow the usual approximation, equation (16), for the lateral size of the beam. In experimental practice the laser is often focussed in a point several centimeters beyond the crystal. As a result the shape of the laser beam within the crystal is conical, rather than cylindrical. The net effect is that the light emitted from differents points of the crystal is focussed in a point placed at a finite distance of the crystal, where the detector is placed. In our calculation we will ignore this complication and consequently the emitted signal and idler beams are taken as consisting of parallel rays.

By using the well-known retarded solution of the inhomogeneous wave equation, the radiated field to first order can be obtained from (14) in the following way:

$$E_1(\mathbf{r},t) = -\frac{1}{4\pi}\int_{\Omega} \text{d}^3\mathbf{r}'\,\frac{S_1(\mathbf{r}', t - \frac{|\mathbf{r}-\mathbf{r}'|}{c'})}{|\mathbf{r}-\mathbf{r}'|}, \quad (17)$$

where the integration is carried over the volume $\Omega$ of the crystal. $c'$ is the speed of light (as a function of the frequency) in the non linear medium and

$$S_1 \equiv \frac{\partial^2(2E_{\text{ZP}}E_{\text{laser}} + E_{\text{laser}}^2)}{\partial t^2} \quad (18)$$

is the source of the field. It, and therefore $E_1$, contains terms of frequencies $\omega_0 - \omega_{\mathbf{k}}$, $\omega_{\mathbf{k}} - \omega_0$, $\omega_0 + \omega_{\mathbf{k}}$, $2\omega_0$ and 0. let us consider them briefly.

(i) The terms of frequency $2\omega_0$ (second harmonic generation) and frequency 0 (rectification of the input laser field) come from $E_{\text{laser}}^2$. As is usual in PDC experiments, we shall deal only with the part of the spectrum which contains oscillatory terms of frequency lower than $\omega_0$, so we can ignore them.

The rest of the terms come from $2E_{\text{ZP}}E_{\text{laser}}$. They are the following: (ii) a term of frequency $\omega_0 - \omega_{\mathbf{k}}$ ($\omega_0 > \omega_{\mathbf{k}}$) that we shall call $E_1^{\text{PDC}}$; (iii) a term of frequency $\omega_{\mathbf{k}} - \omega_0$ ($\omega_{\mathbf{k}} > \omega_0$). This term is a down-conversion of the zero point field (ZPF) and hence we shall call it $E_1^{\text{ZDC}}$. In order to keep its frequencies lower than $\omega_0$, $\omega_{\mathbf{k}}$ must also fulfill $2\omega_0 > \omega_{\mathbf{k}}$. These two terms can be written together as

$$E_1^{\text{DC}}(\mathbf{r},t) \equiv E_1^{\text{PDC}}(\mathbf{r},t) + E_1^{\text{ZDC}}(\mathbf{r},t) = \frac{\text{i}}{2\pi}\sum_{\mathbf{k}}\left(\frac{\hbar\omega}{\epsilon_0 L_0^3}\right)^{\frac{1}{2}} \alpha_{\mathbf{k}}(\omega_0-\omega)^2 \text{e}^{-\text{i}(\omega-\omega_0)t} \times \int_{\Omega}\text{d}^3\mathbf{r}'\,\frac{V(\mathbf{r}')\text{e}^{\text{i}(\mathbf{k}-\mathbf{k}_0)\cdot\mathbf{r}'}\text{e}^{\text{i}(\omega-\omega_0)\frac{|\mathbf{r}-\mathbf{r}'|}{c_{\omega_0-\omega}}}}{|\mathbf{r}-\mathbf{r}'|} + \text{c.c}\ ; \quad 0 < \omega < 2\omega_0, \quad (19)$$

where, for $0 < \omega < \omega_0$ it represents $E_1^{\text{PDC}}$, and for $\omega_0 < \omega < 2\omega_0$, $E_1^{\text{ZDC}}$. We have simplified the notation writing $\omega_{\mathbf{k}} \equiv \omega$. On the other hand, we have put $c_{\omega_0-\omega} \equiv c(\omega_0 - \omega)$, because the components of the radiated field travel with different velocities inside the crystal. Here we have made the customary assumption of considering the crystal embedded in a linear medium with the same dispersion [14].

(iv) Finally, the term of frequency $\omega_0 + \omega_{\mathbf{k}}$. This is an up-conversion of the ZPF – we shall call it $E_1^{\text{ZUC}}$–; its frequencies are always greater than $\omega_0$ and therefore we ignore its contribution to $E_1$. Nevertheless it plays an important role in $E_2$ and we must write its expression:

$$E_1^{\text{ZUC}}(\mathbf{r},t) = \frac{\text{i}}{2\pi}\sum_{\mathbf{k}}\left(\frac{\hbar\omega}{\epsilon_0 L_0^3}\right)^{\frac{1}{2}} \alpha_{\mathbf{k}}(\omega_0+\omega)^2 \text{e}^{-\text{i}(\omega_0+\omega)t} \times \int_{\Omega}\text{d}^3\mathbf{r}'\,\frac{V(\mathbf{r}')\text{e}^{\text{i}(\mathbf{k}_0+\mathbf{k})\cdot\mathbf{r}'}\text{e}^{\text{i}(\omega+\omega_0)\frac{|\mathbf{r}-\mathbf{r}'|}{c_{\omega_0+\omega}}}}{|\mathbf{r}-\mathbf{r}'|} + \text{c.c}, \quad (20)$$

where $c_{\omega_0+\omega} \equiv c(\omega_0+\omega)$.

In the far field approximation

$$\frac{1}{|\mathbf{r}-\mathbf{r}'|} \approx \frac{1}{r}\,; \quad |\mathbf{r}-\mathbf{r}'| \approx r\left(1-\frac{\mathbf{r}\cdot\mathbf{r}'}{r^2}\right), \tag{21}$$

equations (19, 20) read

$$E_1^{\rm DC}(\mathbf{r},t) = \frac{\rm i}{2\pi r}\sum_{\mathbf{k}}\left(\frac{\hbar\omega}{\epsilon_0 L_0^3}\right)^{\frac{1}{2}} \times \alpha_{\mathbf{k}}(\omega_0-\omega)^2 {\rm e}^{-{\rm i}(\omega-\omega_0)t}{\rm e}^{{\rm i}(\omega-\omega_0)\frac{r}{c_{\omega_0-\omega}}} \times \int_\Omega {\rm d}^3\mathbf{r}'\, V(\mathbf{r}'){\rm e}^{{\rm i}(\mathbf{k}-\mathbf{k}_0)\cdot\mathbf{r}'}{\rm e}^{-{\rm i}(\omega-\omega_0)\frac{\mathbf{r}\cdot\mathbf{r}'}{rc_{\omega_0-\omega}}} + {\rm c.c}\,; \quad 0<\omega<2\omega_0, \tag{22}$$

$$E_1^{\rm ZUC}(\mathbf{r},t) = \frac{\rm i}{2\pi r}\sum_{\mathbf{k}}\left(\frac{\hbar\omega}{\epsilon_0 L_0^3}\right)^{\frac{1}{2}} \times \alpha_{\mathbf{k}}(\omega_0+\omega)^2 {\rm e}^{-{\rm i}(\omega_0+\omega)t}{\rm e}^{{\rm i}(\omega_0+\omega)\frac{r}{c_{\omega_0+\omega}}} \times \int_\Omega {\rm d}^3\mathbf{r}'\, V(\mathbf{r}'){\rm e}^{{\rm i}(\mathbf{k}_0+\mathbf{k})\cdot\mathbf{r}'}{\rm e}^{-{\rm i}(\omega+\omega_0)\frac{\mathbf{r}\cdot\mathbf{r}'}{rc_{\omega_0+\omega}}} + {\rm c.c.} \tag{23}$$

We will now calculate the second-order field. From (15), we have

$$E_2(\mathbf{r},t) = -\frac{1}{4\pi}\int_\Omega {\rm d}^3\mathbf{r}''\,\frac{S_2(\mathbf{r}'',t-\frac{|\mathbf{r}-\mathbf{r}''|}{c'})}{|\mathbf{r}-\mathbf{r}''|}, \tag{24}$$

where

$$S_2 \equiv \frac{\partial^2(2E_1E_{\rm laser})}{\partial t^2}\,\cdot \tag{25}$$

Therefore the frequencies that appear are in principle those of $E_1$ plus or minus $\omega_0$. Let us analyse the contributions of $E_1$ ($E_1^{\rm PDC}, E_1^{\rm ZDC}$ and $E_1^{\rm ZUC}$) that induce frequencies lower than $\omega_0$ in $E_2$:

(i) from $E_1^{\rm PDC}$, the frequencies will be $\omega_0-\omega\pm\omega_0$. Remembering that $\omega<\omega_0$ − see discussion above equation (19) − the "+" sign can never give a contribution of frequency lower than $\omega_0$. Then only the contribution with "−" sign is relevant;

(ii) $E_1^{\rm ZDC}$ gives frequencies $\omega-\omega_0\pm\omega_0$, where we had $\omega>\omega_0$. Therefore the "+" sign does not contribute, and the "−" sign contributes only if $\omega<3\omega_0$;

(iii) finally, $E_1^{\rm ZUC}$ gives $\omega+\omega_0\pm\omega_0$ without restriction, in principle, for $\omega$. Hence, only the contribution with "−" sign must be considered in case that $\omega<\omega_0$.

The former three terms will be called $E_2^{\rm PDC}, E_2^{\rm ZDC}$ and $E_2^{\rm ZUC}$ respectively. To evaluate them in the far field approximation we shall use the expressions (19, 20) for the first-order field, together with equations (16, 21, 24, 25). After some easy algebra we obtain:

$$E_2^{\rm PDC}(\mathbf{r},t) = \frac{\rm i}{4\pi^2 r}\sum_{\mathbf{k}}\left(\frac{\hbar\omega}{\epsilon_0 L_0^3}\right)^{\frac{1}{2}}\alpha_{\mathbf{k}}(\omega_0-\omega)^2\omega^2 \times {\rm e}^{-{\rm i}\omega t}{\rm e}^{{\rm i}\omega\frac{r}{c_\omega}}\int_\Omega {\rm d}^3\mathbf{r}'\int_\Omega {\rm d}^3\mathbf{r}''\,V(\mathbf{r}')V(\mathbf{r}'') \times \frac{{\rm e}^{{\rm i}\mathbf{k}_0\cdot(\mathbf{r}''-\mathbf{r}')}{\rm e}^{{\rm i}\mathbf{k}\cdot\mathbf{r}'}{\rm e}^{-{\rm i}\omega\frac{\mathbf{r}\cdot\mathbf{r}''}{rc_\omega}}\,{\rm e}^{{\rm i}(\omega-\omega_0)\frac{|\mathbf{r}'-\mathbf{r}''|}{c_{\omega_0-\omega}}}}{|\mathbf{r}''-\mathbf{r}'|} + {\rm c.c}, \tag{26}$$

$$E_2^{\rm ZUC}(\mathbf{r},t) = \frac{\rm i}{4\pi^2 r}\sum_{\mathbf{k}}\left(\frac{\hbar\omega}{\epsilon_0 L_0^3}\right)^{\frac{1}{2}}\alpha_{\mathbf{k}}(\omega_0+\omega)^2\omega^2 \times {\rm e}^{-{\rm i}\omega t}{\rm e}^{{\rm i}\omega\frac{r}{c_\omega}}\int_\Omega {\rm d}^3\mathbf{r}'\int_\Omega {\rm d}^3\mathbf{r}''\,V(\mathbf{r}')V(\mathbf{r}'') \times \frac{{\rm e}^{{\rm i}\mathbf{k}_0\cdot(\mathbf{r}'-\mathbf{r}'')}{\rm e}^{{\rm i}\mathbf{k}\cdot\mathbf{r}'}{\rm e}^{-{\rm i}\omega\frac{\mathbf{r}\cdot\mathbf{r}''}{rc_\omega}}\,{\rm e}^{{\rm i}(\omega+\omega_0)\frac{|\mathbf{r}'-\mathbf{r}''|}{c_{\omega_0+\omega}}}}{|\mathbf{r}''-\mathbf{r}'|} + {\rm c.c}, \tag{27}$$

$$E_2^{\rm ZDC}(\mathbf{r},t) = \frac{\rm i}{4\pi^2 r}\sum_{\mathbf{k}}\left(\frac{\hbar\omega}{\epsilon_0 L_0^3}\right)^{\frac{1}{2}}\alpha_{\mathbf{k}}(\omega_0-\omega)^2(\omega-2\omega_0)^2 \times {\rm e}^{-{\rm i}(\omega-2\omega_0)t}{\rm e}^{{\rm i}(\omega-2\omega_0)\frac{r}{c_{\omega-2\omega_0}}}\int_\Omega {\rm d}^3\mathbf{r}'\int_\Omega {\rm d}^3\mathbf{r}''\,V(\mathbf{r}')V(\mathbf{r}'') \times \frac{{\rm e}^{-{\rm i}\mathbf{k}_0\cdot(\mathbf{r}'+\mathbf{r}'')}{\rm e}^{{\rm i}\mathbf{k}\cdot\mathbf{r}'}{\rm e}^{-{\rm i}(\omega-2\omega_0)\frac{\mathbf{r}\cdot\mathbf{r}''}{rc_{\omega-2\omega_0}}}\,{\rm e}^{{\rm i}(\omega-\omega_0)\frac{|\mathbf{r}'-\mathbf{r}''|}{c_{\omega_0-\omega}}}}{|\mathbf{r}''-\mathbf{r}'|} + {\rm c.c.} \tag{28}$$

## 4 Field autocorrelation

This section is devoted to the calculation of the autocorrelation function of the field, which is necessary for the study of the spectrum. To start with, let us consider the expression of the field corresponding to frequencies less than $\omega_0$:

$$E = E_{\rm ZP}+\beta(E_1^{\rm PDC}+E_1^{\rm ZDC})+\beta^2(E_2^{\rm PDC}+E_2^{\rm ZUC}+E_2^{\rm ZDC}), \tag{29}$$

where, for notation simplicity, we have not written the dependence on position and time.

The autocorrelation, at a given position and different times $t$ and $t'$, is given by:

$$\begin{aligned}\langle E(t)E(t')\rangle &= \langle E_{\rm ZP}(t)E_{\rm ZP}(t')\rangle \\ &+ \beta[\langle E_{\rm ZP}(t)E_1^{\rm PDC}(t')\rangle + \langle E_{\rm ZP}(t')E_1^{\rm PDC}(t)\rangle] \\ &+ \beta[\langle E_{\rm ZP}(t)E_1^{\rm ZDC}(t')\rangle + \langle E_{\rm ZP}(t')E_1^{\rm ZDC}(t)\rangle] \\ &+ \beta^2[\langle E_1^{\rm PDC}(t)E_1^{\rm PDC}(t')\rangle + \langle E_{\rm ZP}(t)E_2^{\rm PDC}(t')\rangle \\ &+ \langle E_{\rm ZP}(t')E_2^{\rm PDC}(t)\rangle + \langle E_1^{\rm ZDC}(t)E_1^{\rm ZDC}(t')\rangle \\ &+ \langle E_{\rm ZP}(t)E_2^{\rm ZUC}(t')\rangle + \langle E_{\rm ZP}(t')E_2^{\rm ZUC}(t)\rangle \\ &+ \langle E_1^{\rm PDC}(t)E_1^{\rm ZDC}(t')\rangle + \langle E_1^{\rm PDC}(t')E_1^{\rm ZDC}(t)\rangle \\ &+ \langle E_{\rm ZP}(t)E_2^{\rm ZDC}(t')\rangle + \langle E_{\rm ZP}(t')E_2^{\rm ZDC}(t)\rangle].\end{aligned} \tag{30}$$

The terms $\langle E_1^{\rm PDC} E_1^{\rm ZDC}\rangle$ are null because the two fields involved contain modes of different frequencies, hence being uncorrelated. In principle, the rest of correlations in this equation should be calculated to get the spectrum of the radiation. Nevertheless, because of the spectrum is related to the Fourier transform of the field, only a few terms of the above expression are relevant, namely those that vary with $t$ and $t'$ as a function of the difference $t-t'$, because the other kind of time dependence that appears in (30) will average to zero. By taking into account the expressions of the different contributions of the field calculated in Section 3, and (11), it can be easily seen that only the correlations corresponding to the first, fourth and fifth lines of (30) will contribute. The rest of the terms will not give a net contribution to the spectrum. Hence we shall retain, from equation (30), only the relevant correlations. We have:

$$\begin{aligned}\langle E(t)E(t')\rangle &= \langle E_{\rm ZP}(t)E_{\rm ZP}(t')\rangle + \beta^2[\langle E_1^{\rm PDC}(t)E_1^{\rm PDC}(t')\rangle \\ &+ \langle E_{\rm ZP}(t)E_2^{\rm PDC}(t')\rangle + \langle E_{\rm ZP}(t')E_2^{\rm PDC}(t)\rangle \\ &+ \langle E_1^{\rm ZDC}(t)E_1^{\rm ZDC}(t')\rangle + \langle E_{\rm ZP}(t)E_2^{\rm ZUC}(t')\rangle \\ &+ \langle E_{\rm ZP}(t')E_2^{\rm ZUC}(t)\rangle]. \qquad (31)\end{aligned}$$

The first term on the right-hand side corresponds to pure zeropoint field and we will not calculate it because it is irrelevant for our purposes (see first paragraph of Sect. 5).

Consider the contribution of the first-order parametric down-converted term. By using (22, 11), we have

$$\begin{aligned}\langle E_1^{\rm PDC}(t)E_1^{\rm PDC}(t')\rangle &= \langle E_1^{\rm PDC(+)}(t)E_1^{\rm PDC(-)}(t')\rangle + {\rm c.c} \\ &= \frac{1}{4\pi^2 r^2\epsilon_0}\frac{1}{L_0^3}\sum_{\mathbf{k}}\hbar\omega(\omega_0-\omega)^4\cos[(\omega_0-\omega)(t-t')] \\ &\times\left|\int_\Omega {\rm d}^3\mathbf{r}'\,V(\mathbf{r}'){\rm e}^{{\rm i}(\mathbf{k}-\mathbf{k}_0)\cdot\mathbf{r}'}{\rm e}^{-{\rm i}(\omega-\omega_0)\frac{\mathbf{r}\cdot\mathbf{r}'}{rc_{\omega_0-\omega}}}\right|^2 \\ &= \frac{1}{4\pi^2 r^2\epsilon_0}\frac{1}{L_0^3}\sum_{\mathbf{k}}\hbar\omega(\omega_0-\omega)^4\cos[(\omega_0-\omega)(t-t')] \\ &\times\int_\Omega {\rm d}^3\mathbf{r}'\,V(\mathbf{r}'){\rm e}^{{\rm i}(\mathbf{k}-\mathbf{k}_0)\cdot\mathbf{r}'}{\rm e}^{-{\rm i}(\omega-\omega_0)\frac{\mathbf{r}\cdot\mathbf{r}'}{rc_{\omega_0-\omega}}} \\ &\times\int_\Omega {\rm d}^3\mathbf{r}''\,V(\mathbf{r}''){\rm e}^{-{\rm i}(\mathbf{k}-\mathbf{k}_0)\cdot\mathbf{r}''}{\rm e}^{{\rm i}(\omega-\omega_0)\frac{\mathbf{r}\cdot\mathbf{r}''}{rc_{\omega_0-\omega}}}, \qquad (32)\end{aligned}$$

where $\omega<\omega_0$. Replacing the sum by an integral, *i.e.* making $\sum_{\mathbf{k}}/L_0^3 \to \int {\rm d}^3k/(2\pi)^3$, and changing to polar spherical coordinates $(k_x,k_y,k_z)\to(\omega,\theta,\psi)$, $\theta$ being the angle between $\mathbf{k}$ and $\mathbf{r}'-\mathbf{r}''$, we obtain, after performing the integration over $\theta$ and $\psi$:

$$\begin{aligned}\langle E_1^{\rm PDC}(t)E_1^{\rm PDC}(t')\rangle &= \frac{2\hbar}{(2\pi)^4 r^2\epsilon_0} \\ &\times\int_0^{\omega_0}{\rm d}\omega\,(\omega_0-\omega)^4\frac{\omega^3}{c_\omega^3}\cos[(\omega_0-\omega)(t-t')] \\ &\times\int_\Omega {\rm d}^3\mathbf{r}'\int_\Omega {\rm d}^3\mathbf{r}''\,V(\mathbf{r}')V(\mathbf{r}''){\rm e}^{-{\rm i}(\omega-\omega_0)\frac{\mathbf{r}\cdot(\mathbf{r}'-\mathbf{r}'')}{rc_{\omega_0-\omega}}} \\ &\times{\rm e}^{-{\rm i}\mathbf{k}_0\cdot(\mathbf{r}'-\mathbf{r}'')}{\rm sinc}\left[\frac{\omega|\mathbf{r}'-\mathbf{r}''|}{c_\omega}\right]. \qquad (33)\end{aligned}$$

In order to calculate the contribution to the autocorrelation of the second-order parametric down-converted radiation we shall use equations (10, 26). By taking into account (11), we arrive to the following expression:

$$\begin{aligned}&\langle E_{\rm ZP}(t)E_2^{\rm PDC}(t')\rangle + \langle E_{\rm ZP}(t')E_2^{\rm PDC}(t)\rangle \\ &= \frac{1}{4\pi^2 r\epsilon_0}\frac{1}{L_0^3}\sum_{\mathbf{k}}\hbar\omega^3(\omega_0-\omega)^2\cos[\omega(t-t')]{\rm e}^{{\rm i}\frac{\omega r}{c_\omega}} \\ &\times\int_\Omega {\rm d}^3\mathbf{r}'\int_\Omega {\rm d}^3\mathbf{r}''\,V(\mathbf{r}')V(\mathbf{r}'') \\ &\times\frac{{\rm e}^{-{\rm i}\mathbf{k}_0\cdot(\mathbf{r}'-\mathbf{r}'')}{\rm e}^{{\rm i}\mathbf{k}\cdot(\mathbf{r}'-\mathbf{r})}{\rm e}^{-{\rm i}\omega\frac{\mathbf{r}\cdot\mathbf{r}''}{rc_\omega}}{\rm e}^{{\rm i}(\omega-\omega_0)\frac{|\mathbf{r}'-\mathbf{r}''|}{c_{\omega_0-\omega}}}}{|\mathbf{r}''-\mathbf{r}'|} + {\rm c.c.}, \qquad (34)\end{aligned}$$

where $\omega<\omega_0$. Replacing the sum by an integral, and changing to polar spherical coordinates $(k_x,k_y,k_z)\to(\omega,\theta,\psi)$, $\theta$ being now the angle between $\mathbf{k}$ and $\mathbf{r}'-\mathbf{r}''$, we get, after the integration over $\theta$ and $\psi$:

$$\begin{aligned}&\langle E_{\rm ZP}(t)E_2^{\rm PDC}(t')\rangle + \langle E_{\rm ZP}(t')E_2^{\rm PDC}(t)\rangle \\ &= \frac{\hbar}{\pi r(2\pi)^3\epsilon_0}\int_0^{\omega_0}{\rm d}\omega\,(\omega_0-\omega)^2\frac{\omega^5}{c_\omega^3}\cos[\omega(t-t')] \\ &\times\int_\Omega {\rm d}^3\mathbf{r}'\int_\Omega {\rm d}^3\mathbf{r}''\,V(\mathbf{r}')V(\mathbf{r}'') \\ &\times\frac{{\rm e}^{-{\rm i}\mathbf{k}_0\cdot(\mathbf{r}'-\mathbf{r}'')}{\rm e}^{-{\rm i}\omega\frac{\mathbf{r}\cdot\mathbf{r}''}{rc_\omega}}{\rm e}^{{\rm i}(\omega-\omega_0)\frac{|\mathbf{r}'-\mathbf{r}''|}{c_{\omega_0-\omega}}}}{|\mathbf{r}''-\mathbf{r}'|} \\ &\times{\rm e}^{{\rm i}\frac{\omega r}{c_\omega}}{\rm sinc}\left[\frac{\omega|\mathbf{r}'-\mathbf{r}|}{c_\omega}\right] + {\rm c.c.} \qquad (35)\end{aligned}$$

We now use the far field approximations (21) in order to express the last term of equation (35) in the following way:

$${\rm e}^{{\rm i}\frac{\omega r}{c_\omega}}{\rm sinc}\left\{\frac{\omega|\mathbf{r}'-\mathbf{r}|}{c_\omega}\right\} \approx \frac{c_\omega}{2{\rm i}r\omega}[-{\rm e}^{{\rm i}\omega\frac{\mathbf{r}\cdot\mathbf{r}'}{rc_\omega}} + {\rm e}^{2{\rm i}\frac{\omega r}{c_\omega}}{\rm e}^{-{\rm i}\omega\frac{\mathbf{r}\cdot\mathbf{r}'}{rc_\omega}}]. \qquad (36)$$

Because of the distance $r$ is much larger than the typical wavelength of light, the second term of (36) is rapidly oscillating as a function of $\omega$, and its net contribution to the integral is negligeable. After some easy algebra in which some change of variables is made in order to obtain a more compact expression, we arrive at the following result:

$$\begin{aligned}&\langle E_{\rm ZP}(t)E_2^{\rm PDC}(t')\rangle + \langle E_{\rm ZP}(t')E_2^{\rm PDC}(t)\rangle \\ &= \frac{2\hbar}{(2\pi)^4 r^2\epsilon_0}\int_0^{\omega_0}{\rm d}\omega\,(\omega_0-\omega)^4\frac{\omega^3}{c_\omega^3}\frac{c_\omega^2}{c_{\omega_0-\omega}^2}\cos[(\omega_0-\omega)(t-t')] \\ &\times\int_\Omega {\rm d}^3\mathbf{r}'\int_\Omega {\rm d}^3\mathbf{r}''\,V(\mathbf{r}')V(\mathbf{r}''){\rm e}^{-{\rm i}(\omega-\omega_0)\frac{\mathbf{r}\cdot(\mathbf{r}'-\mathbf{r}'')}{rc_{\omega_0-\omega}}} \\ &\times{\rm e}^{-{\rm i}\mathbf{k}_0\cdot(\mathbf{r}'-\mathbf{r}'')}{\rm sinc}\left[\frac{\omega|\mathbf{r}'-\mathbf{r}''|}{c_\omega}\right]. \qquad (37)\end{aligned}$$

Now it is time to analyze the results obtained up to here and resumed in equations (33, 37). In previous works [9], when the theoretical treatment was based on the standard Hamiltonian model, the contribution to the PDC phenomena coming from (33, 37) were equal. In the present and more detailed analysis we see that there is a small difference between them, arising from the ratio $c_\omega^2/c_{\omega_0-\omega}^2$. This factor, although close to unity, is exactly one only in the degenare case, $\omega = \omega_0/2$.

Let us finally consider the rest of contributions of the autocorrelation. Similar calculations in which (22, 27) are used, led to the following expressions:

$$\begin{aligned}\langle E_1^{\mathrm{ZDC}}(t)E_1^{\mathrm{ZDC}}(t')\rangle &= \frac{2\hbar}{(2\pi)^4 r^2\epsilon_0}\int_0^{\omega_0}\mathrm{d}\omega\,(\omega_0+\omega)^3\frac{\omega^4}{c_{\omega_0+\omega}^3}\cos[\omega(t-t')] \\ &\times\int_\Omega \mathrm{d}^3\mathbf{r}'\int_\Omega \mathrm{d}^3\mathbf{r}''\,V(\mathbf{r}')V(\mathbf{r}'')\mathrm{e}^{\mathrm{i}\omega\frac{\mathbf{r}\cdot(\mathbf{r}'-\mathbf{r}'')}{rc_\omega}} \\ &\times \mathrm{e}^{\mathrm{i}\mathbf{k}_0\cdot(\mathbf{r}'-\mathbf{r}'')}\mathrm{sinc}\left[\frac{(\omega_0+\omega)|\mathbf{r}'-\mathbf{r}''|}{c_{\omega_0+\omega}}\right], \end{aligned}\quad (38)$$

$$\begin{aligned}\langle E_{\mathrm{ZP}}(t)E_2^{\mathrm{ZUC}}(t')\rangle &+ \langle E_{\mathrm{ZP}}(t')E_2^{\mathrm{ZUC}}(t)\rangle \\ &= -\frac{2\hbar}{(2\pi)^4 r^2\epsilon_0}\int_0^{\omega_0}\mathrm{d}\omega\,(\omega_0+\omega)^3\frac{\omega^4}{c_{\omega_0+\omega}^3}\frac{c_{\omega_0+\omega}^2}{c_\omega^2}\cos[\omega(t-t')] \\ &\times\int_\Omega \mathrm{d}^3\mathbf{r}'\int_\Omega \mathrm{d}^3\mathbf{r}''\,V(\mathbf{r}')V(\mathbf{r}'')\mathrm{e}^{\mathrm{i}\omega\frac{\mathbf{r}\cdot(\mathbf{r}'-\mathbf{r}'')}{rc_\omega}} \\ &\times \mathrm{e}^{\mathrm{i}\mathbf{k}_0\cdot(\mathbf{r}'-\mathbf{r}'')}\mathrm{sinc}\left[\frac{(\omega_0+\omega)|\mathbf{r}'-\mathbf{r}''|}{c_{\omega_0+\omega}}\right]. \end{aligned}\quad (39)$$

The results obtained in equations (38, 39) show that these two terms are very similar, but with opposite sign. They do not cancel each other because of the factor $-c_{\omega_0+\omega}^2/c_\omega^2$ in (39). This difference will give rise to a net contribution to the intensity spectrum coming from the up and down conversion of the ZPF. In this way we predict a new phenomena which is not predicted in the standard treatments. A similar result was obtained by one of us from a different approach and we refer to that work for details [17]. We shall not develop this point here because a correct treatment of it can only be performed by taking into account the polarization of the electric field, and not in the present scalar approximation. For this reason the rest of the paper will be devoted only to the study of the spectral properties of the PDC radiation.

## 5 Analysis of the PDC spectrum

The quantum theory of detection in the Wigner representation shows that the single detection rate at the position $\mathbf{r}$ and time $t$ is proportional to the quantity (for more details see Refs. [9,11])

$$P_1(\mathbf{r},t) \propto \langle I(\mathbf{r},t) - I_0(\mathbf{r})\rangle, \quad (40)$$

$I \propto |E|^2$ being the total intensity of light and $I_0 \propto \langle |E_{\mathrm{ZP}}|^2\rangle$ the average intensity of the zeropoint field. $I$ and $I_0$ are not well-defined if we do not specify which is the relevant frequency range involved in the sum over $\mathbf{k}$ (that range is essentially defined by the detectors). However, $I - I_0$ is well-defined because for all modes which do not take part in the detection that difference is zero. Because the substration of the zeropoint intensity plays an essential role in the detection process, our analysis of the spectral properties of the radiated field will start from the following expression:

$$\begin{aligned} I(\nu,\mathbf{r}) = c\epsilon_0\int_{-\infty}^{+\infty}(\langle E(\mathbf{r},t)E(\mathbf{r},t')\rangle - \langle E_{\mathrm{ZP}}(\mathbf{r},t)E_{\mathrm{ZP}}(\mathbf{r},t')\rangle) \\ \times\cos[\nu(t-t')]\,\mathrm{d}(t-t'), \end{aligned}\quad (41)$$

where $c$ is a typical value of the velocity of light in the nonlinear medium, which has been introduced in order to get $I(\nu,\mathbf{r})$ in units of energy per unit area and unit frequency interval.

By taking into account the relations between the different terms of the autocorrelation, for the different parts of the PDC radiated field, we have

$$\begin{aligned} I^{\mathrm{PDC}}(\nu,\mathbf{r}) &= c\beta^2\epsilon_0\left(1+\frac{c_{\omega_0-\nu}^2}{c_\nu^2}\right) \\ &\times\int_{-\infty}^{+\infty}\langle E_1^{\mathrm{PDC}}(\mathbf{r},t)E_1^{\mathrm{PDC}}(\mathbf{r},t')\rangle\cos[\nu(t-t')]\,\mathrm{d}(t-t'), \end{aligned}\quad (42)$$

where $\langle E_1^{\mathrm{PDC}}(\mathbf{r},t)E_1^{\mathrm{PDC}}(\mathbf{r},t')\rangle$ was obtained in (32).

The integral on the volume of the non linear crystal that appears there is straightforward. By substituting (32) into (42) and performing the integral in $t-t'$ we arrive at

$$\begin{aligned} I^{\mathrm{PDC}}(\nu,\mathbf{r}) &= \pi\beta^2\Big(1+\frac{c_{\omega_0-\nu}^2}{c_\nu^2}\Big)F(r)\frac{1}{L_0^3} \\ &\times\sum_{\mathbf{k}}\omega(\omega_0-\omega)^4\delta(\omega_0-\omega-\nu) \\ &\times\exp\Big\{-R^2[(\omega_0-\omega)\frac{x}{rc_{\omega_0-\omega}}+k_x]^2 \\ &\quad -R^2[(\omega_0-\omega)\frac{y}{rc_{\omega_0-\omega}}+k_y]^2\Big\} \\ &\times\mathrm{sinc}^2\left\{\frac{L}{2}[(\omega_0-\omega)\frac{z}{rc_{\omega_0-\omega}}+k_z-k_0]\right\}, \end{aligned}\quad (43)$$

where

$$F(r) = c\hbar\left(\frac{R^2LV_0}{r}\right)^2, \quad (44)$$

$L$ being the length of the crystal and $\mathbf{r} = x\mathbf{u}_x + y\mathbf{u}_y + z\mathbf{u}_z$. From now on we shall take $\mathbf{r}$ in the plane $OXZ$, without a loss of generality, *i.e.* $y = 0$. Passing to the continuum and changing to polar spherical coordinates

$$I^{\rm PDC}(\nu,x,z)=\frac{\beta^2F(r)}{8\pi^{3/2}}(1+\frac{c^2_{\omega_0-\nu}}{c^2_\nu})\int_{-1}^{1}{\rm d}(\cos\theta)\int_0^{\omega_0}\omega^3\,{\rm d}\omega\delta(\omega_0-\omega-\nu)(\omega_0-\omega)^4$$
$$\times\frac{\exp\left\{-R^2[(\omega_0-\omega)^2\frac{x^2}{r^2c^2_{\omega_0-\omega}}+\frac{\omega^2}{c^2_\omega}\sin^2\theta-2(\omega_0-\omega)\frac{x}{rc_{\omega_0-\omega}}\frac{\omega}{c_\omega}\sin\theta]\right\}}{c^3_\omega R\sqrt{\frac{(\omega_0-\omega)x\omega\sin\theta}{rc_\omega c_{\omega_0-\omega}}}}{\rm sinc}^2\left\{\frac{L}{2}[(\omega_0-\omega)\frac{z}{rc_{\omega_0-\omega}}+\frac{\omega}{c_\omega}\cos\theta-k_0]\right\}. \quad (48)$$

$$I^{\rm PDC}(\nu,\alpha,r)=\frac{\beta^2F(r)\nu^4}{4L\pi^{1/2}}\frac{(\omega_0-\nu)^2}{c^2_{\omega_0-\nu}}\left(1+\frac{c^2_{\omega_0-\nu}}{c^2_\nu}\right)\frac{\exp\left\{-R^2\left[\frac{\nu}{c_\nu}\sin\alpha-\frac{(\omega_0-\nu)}{c_{\omega_0-\nu}}\sqrt{1-\frac{c^2_{\omega_0-\nu}}{(\omega_0-\nu)^2}[\frac{\omega_0}{c_0}-\frac{\nu}{c_\nu}\cos\alpha]^2}\right]^2\right\}}{R\sqrt{\frac{\nu\sin\alpha(\omega_0-\nu)}{c_\nu c_{\omega_0-\nu}}\sqrt{1-\frac{c^2_{\omega_0-\nu}}{(\omega_0-\nu)^2}[\frac{\omega_0}{c_0}-\frac{\nu}{c_\nu}\cos\alpha]^2}}}. \quad (51)$$

$(k_x,k_y,k_z)\to(\omega,\theta,\psi)$, the polar axis being $\mathbf{u}_z$, we have:

$$I^{\rm PDC}(\nu,x,z)=\frac{\beta^2F(r)}{8\pi^2}(1+\frac{c^2_{\omega_0-\nu}}{c^2_\nu})\times\int_0^{2\pi}{\rm d}\phi\int_{-1}^{1}{\rm d}(\cos\theta)\int_0^{\omega_0}{\rm d}\omega\frac{\omega^3}{c^3_\omega}(\omega_0-\omega)^4\delta(\omega_0-\omega-\nu)$$
$$\times\exp\left\{-R^2[(\omega_0-\omega)^2\frac{x^2}{r^2c^2_{\omega_0-\omega}}+\frac{\omega^2}{c^2_\omega}\sin^2\theta+2(\omega_0-\omega)\frac{x}{rc_{\omega_0-\omega}}\frac{\omega}{c_\omega}\sin\theta\cos\phi]\right\}$$
$$\times{\rm sinc}^2\left\{\frac{L}{2}[(\omega_0-\omega)\frac{z}{rc_{\omega_0-\omega}}+\frac{\omega}{c_\omega}\cos\theta-k_0]\right\}. \quad (45)$$

The integration in $\phi$ is straightforward:

$$\int_{-\pi}^{\pi}{\rm d}\phi\,{\rm e}^{-m\cos\phi}\simeq\left(\frac{2\pi}{m}\right)^{1/2}{\rm e}^m, \quad (46)$$

if $m\gg1$. In our case

$$m=2R^2(\omega_0-\omega)\frac{x}{rc_{\omega_0-\omega}}\frac{\omega}{c_\omega}\sin\theta, \quad (47)$$

which is, roughly speaking, the squared ratio between the radius of the laser and the wavelength of the radiation, much greater than 1, and therefore

*see equation (48) above.*

By performing the integration in $\omega$

$$I^{\rm PDC}(\nu,x,z)=\frac{\beta^2F(r)\nu^4(\omega_0-\nu)^3}{8\pi^{3/2}}(1+\frac{c^2_{\omega_0-\nu}}{c^2_\nu})\times\int_{-1}^{1}{\rm d}(\cos\theta)\frac{\exp\left\{-R^2[\frac{\nu x}{rc_\nu}-\frac{(\omega_0-\nu)}{c_{\omega_0-\nu}}\sin\theta]^2\right\}}{c^3_{\omega_0-\nu}R\sqrt{\frac{\nu x(\omega_0-\nu)\sin\theta}{rc_\nu c_{\omega_0-\nu}}}}$$
$$\times{\rm sinc}^2\left\{\frac{L}{2}[\frac{\nu z}{rc_\nu}+\frac{\omega_0-\nu}{c_{\omega_0-\nu}}\cos\theta-\frac{\omega_0}{c_0}]\right\}. \quad (49)$$

In the former expression there is one Gaussian factor and one sinc squared. These functions are important only for a small range of frequencies. This means that in the case of large values of $R$ or large values of $L$ one of these functions can be approximated by a delta. Let us study these two limiting cases, which correspond to $R$ being much greater or much smaller than $Lx/r$.

### Case I: Long crystal ($L\to\infty$)

If we multiply and divide by $L/2$ and take the limit $L\to\infty$ in the appropriate place we get

$$I^{\rm PDC}(\nu,x,z)=\frac{\beta^2F(r)\nu^4(\omega_0-\nu)^3}{4L\pi^{1/2}c^3_{\omega_0-\nu}}(1+\frac{c^2_{\omega_0-\nu}}{c^2_\nu})\int_{-1}^{1}{\rm d}(\cos\theta)\times\frac{\exp\left\{-R^2[\frac{\nu x}{rc_\nu}-\frac{(\omega_0-\nu)}{c_{\omega_0-\nu}}\sin\theta]^2\right\}}{R\sqrt{\frac{\nu x(\omega_0-\nu)\sin\theta}{rc_\nu c_{\omega_0-\nu}}}}$$
$$\times\delta\left(\frac{\nu z}{rc_\nu}+\frac{\omega_0-\nu}{c_{\omega_0-\nu}}\cos\theta-\frac{\omega_0}{c_0}\right)\cdot \quad (50)$$

Using

$$\int{\rm d}x\,\delta[f(x)]g(x)=\sum_i\frac{g(x_i)}{|f\prime(x_i)|};\quad f(x_i)=0,$$

and making $x/r=\sin\alpha$, $z/r=\cos\alpha$, we obtain

*see equation (51) above.*

By considering typical values of the experimental parameters ($R\approx10^{-3}$ m, $\omega_0\approx10^{15}$ rad s$^{-1}$) we have

$$\left(\frac{R\omega_0}{c}\right)^2\approx10^8, \quad (52)$$

and then, for a given value of $\alpha$,the relevant contributions to the intensity are those coming from frequencies nearly equal to $\nu_\alpha$, the frequency that maximizes $I^{\rm PDC}(\nu,\alpha,r)$. It is the one that makes the exponent of the first term

$$I^{\mathrm{PDC}}(\nu,\alpha,r) = \frac{\beta^2 F(r)\nu^3 c_\nu(\omega_0-\nu)^2}{8\pi R^2 c^2_{\omega_0-\nu}\sin\alpha}\left(1 + \frac{c^2_{\omega_0-\nu}}{c^2_\nu}\right)\frac{\mathrm{sinc}^2\left\{\frac{L}{2}\left[\frac{\nu}{c_\nu}\cos\alpha+\frac{\omega_0-\nu}{c_{\omega_0-\nu}}\sqrt{1-\frac{\nu^2 c^2_{\omega_0-\nu}}{c^2_\nu(\omega_0-\nu)^2}\sin^2\alpha}-\frac{\omega_0}{c_0}\right]\right\}}{\sqrt{1-\frac{\nu^2 c^2_{\omega_0-\nu}}{c^2_\nu(\omega_0-\nu)^2}\sin^2\alpha}}\cdot \quad (59)$$

equals to zero. By taking into account that $x^2+z^2=r^2$ we have the following relation between $\nu_\alpha$ and $\alpha$ for the optimum position of the detector:

$$\cos\alpha = \frac{z}{r} = \left[\frac{\nu_\alpha^2}{c^2_{\nu_\alpha}} - \frac{(\omega_0-\nu_\alpha)^2}{c^2_{\omega_0-\nu_\alpha}} + \frac{\omega_0^2}{c_0^2}\right]\frac{c_0 c_{\nu\alpha}}{2\nu_\alpha\omega_0}\cdot \quad (53)$$

By taking into account (52) we shall express the argument of the exponential factor of (51) to second order in $\nu-\nu_\alpha$ and to zeroth order elsewhere. For simplicity only two velocities of light will be considered, namely the corresponding to the laser $c_0$ and $c_\nu \simeq c_{\omega_0-\nu} \equiv c$. After some easy calculations in which we use equation (44), we arrive at the following expression for the PDC spectrum in the long crystal case:

$$I^{\mathrm{PDC}}(\nu,\alpha,r) = \frac{\hbar R^3 L V_0^2 \beta^2 \nu_\alpha^3(\omega_0-\nu_\alpha)^2}{2\pi^{1/2} r^2 \sin\alpha}\mathrm{e}^{-\frac{(\nu-\nu_\alpha)^2}{2\Delta_\alpha^2}}, \quad (54)$$

where

$$\Delta_\alpha = \frac{c\nu_\alpha\sin\alpha}{\sqrt{2}R\omega_0\left(1-\frac{c}{c_0}\cos\alpha\right)}, \quad (55)$$

and, in the case $c\equiv c_\nu \simeq c_{\omega_0-\nu}$, the relation between $\nu_\alpha$ and $\alpha$ is given by

$$\nu_\alpha = \frac{\omega_0}{2}\frac{1-\frac{c^2}{c_0^2}}{1-\frac{c}{c_0}\cos\alpha}\cdot \quad (56)$$

For instance, if we consider the degenerate case $\nu=\omega_0/2$, $\alpha$ is given by the relation $\cos\alpha = c_{\omega_0/2}/c_0$, and then

$$\Delta_\alpha = \frac{c}{2\sqrt{2}R\sin\alpha}, \quad (57)$$

expression that coincides with the inverse of the correlation time between signal and idler photons (see Eq. (50) of Ref. [12]). This agreement between the coherence time of the signal (or the idler) beam and the cross-correlation time is remarkable and it is the basis of the most relevant properties of PDC (in common language it is expressed by saying that the two partner photons are emitted at the same time).

### Case II: Short crystal ($\mathbf{R}\to\infty$)

Now let us explore the consequences of assuming a very large radius of the pumping: $R\to\infty$. Taking into account that

$$\lim_{R\to\infty} R\mathrm{e}^{-R^2x^2} = \sqrt{\pi}\delta(x), \quad (58)$$

the spectrum is, from (49)

$$I_1^{\mathrm{PDC}}(\nu,x,z) = \frac{\beta^2 F(r)\nu^4(\omega_0-\nu)^3}{8\pi R c^3_{\omega_0-\nu}}\int_{-1}^{1}\mathrm{d}(\cos\theta) \times \frac{\delta\left(\frac{\nu x}{r c_\nu} - \frac{(\omega_0-\nu)}{c_{\omega_0-\nu}}\sin\theta\right)}{R\sqrt{\frac{\nu x(\omega_0-\nu)\sin\theta}{r c_\nu c_{\omega_0-\nu}}}} \times \mathrm{sinc}^2\left\{\frac{L}{2}\left[\frac{\nu z}{r c_\nu}+\frac{\omega_0-\nu}{c_{\omega_0-\nu}}\cos\theta-\frac{\omega_0}{c_0}\right]\right\}\cdot$$

Therefore

*see equation (59) above.*

By considering typical values ($L\approx 10^{-2}$ m, $\omega_0 \approx 10^{15}$ rad s$^{-1}$) we have

$$\frac{L\omega_0}{c}\approx 10^5,$$

and then, for a given value of $\alpha$, the relevant contributions to the intensity are those coming from frequencies nearly equal to $\nu_\alpha$, the frequency that maximizes $I^{\mathrm{PDC}}(\nu,\alpha,r)$. It is the one that makes the argument of the sinc$^2$ equals to zero, and coincides with (53). Now, by expanding to first order in of $\nu-\nu_\alpha$ the argument of the sinc and to zeroth order elsewhere, and by using equation (44) we arrive at the following result:

$$I^{\mathrm{PDC}}(\nu,\alpha,r) = \frac{\hbar L^2 R^2 V_0^2\beta^2\nu_\alpha^3(\omega_0-\nu_\alpha)^2}{4\pi r^2\sin\alpha\sqrt{1-\frac{\nu_\alpha^2}{(\omega_0-\nu_\alpha)^2}\sin^2\alpha}} \times\mathrm{sinc}^2\left(\frac{\nu-\nu_\alpha}{\Delta_\alpha}\right), \quad (60)$$

where

$$\Delta_\alpha = \frac{2c\left(\nu_\alpha\cos\alpha - \frac{c}{c_0}\omega_0\right)}{L\omega_0\left(1-\frac{c}{c_0}\cos\alpha\right)}\cdot \quad (61)$$

Again, if we consider the degenerate case, it can be easily proved that the width of the sinc$^2$ function is of the same order of the quantity $c\cos\alpha/L\sin^2\alpha$, corresponding to the inverse of the correlation time between signal and idler photons (see Eq. (41) of Ref. [12]).

## 6 Discussion: PDC experiments and local realism

This is the sixth of a series of articles devoted to the study of parametric down conversion in the Wigner-function formalism. In the first four [8–11] we started from a Hamiltonian approach and showed that all PDC experiments performed until the end of 1997 may be interpreted within the formalism. It is also possible to interpret more recent experiments; for instance we may predict all the correlations which are actually measured in reported quantum teleportation experiments [18,19]. This simply confirms that the Wigner representation is a valid formulation of quantum optics, fully equivalent to the more common Hilbert space formalism. But the Wigner function provides an intuitive picture of PDC in terms of classical (Maxwell) waves propagating causally in space and time. The only difference with standard classical electrodynamics is the presence of a random electromagnetic radiation (zeropoint field, ZPF) filling the whole space. That radiation may be expanded in plane waves and the probability distribution for the amplitudes is given by equation (9). A crucial point for the possibility of a classical picture is the fact that the Wigner distribution is, in the case of PDC, positive definite.

In our last two papers on PDC ([12] and the present one) we use an approach more fundamental than the model Hamiltonian; in fact we start from the quantized electromagnetic field in the nonlinear crystal. As a result we have shown that the production of PDC radiation is formally equivalent to the classical polarization of the crystal by the combined action of the laser pump and the zeropoint field, which causes the crystal to reemit radiation. We have studied the stochastic properties of that radiation by calculating the cross-correlation of signal and idler [12] and their autocorrelations (in the present paper). In both cases we have used a standard perturbative expansion of the retarded (causal) Green's function. It is interesting that the intensity of the cross-correlation (calculated in Ref. [12]) may be obtained to second order by just calculating the field to first order, whilst we need the field to second order for the autocorrelation, so that the latter is substantially more difficult than the former. From these correlations we have been able to derive the correlation time of what in standard quantum language are called "signal and idler photons", and also the spectrum of the emitted radiation, all in terms of the parameters of the crystal and the pumping beam.

It is remarkable that in the Wigner-function formalism there is no trace of "photons"; we have just correlated waves. Effects like the strong correlation between colour and direction of emission, which in the standard (Hilbert space) approach appears as derived from the conservation of energy and momentum in the process of splitting of a laser *photon*, are, in the Wigner-function approach, a consequence of energy and momentum conservation of the *field*. And the directionality appears, as is typical in wave optics, due to the interference (constructive in a specific direction, destructive otherwise) of the radiation emitted from all points of the crystal. Then it is not surprising that the bigger the crystal the better the directionality of the emitted radiation, as shown in [12] and the present paper, a relation which is less clear in the standard (Hilbert space) approach. Even effects so typically quantal as the "photon entanglement", appear in the Wigner-function formalism just as correlations which involve both the zeropoint and the superimposed radiation, whilst normal correlation involves only the radiation above the ZPF (see Sect. 5 of Ref. [11]).

The above discussion leads us to an apparently paradoxical situation. We have arrived at a purely wave (classical) picture of PDC and, nevertheless, PDC is the phenomenon most frequently claimed to exhibit non-classical aspects of light, like teleportation or violation of the Bell inequality. Clearly this situation requires a deeper study, which we now make.

We begin by substituting a better expression for the somewhat ambiguous word "classical". We propose "local realist" or "local hidden variables" (LHV) model in the well defined sense given by Bell [20]. We shall call local realist any theory where the probability of a coincidence photocount may be obtained from the expression

$$P_{12} = \int \rho(\lambda) P_1(\lambda, \phi_1) P_2(\lambda, \phi_2) \mathrm{d}\lambda\,, \quad \rho(\lambda) \geq 0,$$
$$\int \rho(\lambda)\mathrm{d}\lambda = 1; \quad 0 \leq P_1(\lambda, \phi_1), \quad P_2(\lambda, \phi_2) \leq 1\,, \quad (62)$$

and similar expressions for single counts, triple coincidence counts, etc. Here $\lambda$ represent the hidden variables and $\phi_1$, $\phi_2$ are controllable parameters (*e.g.* angles of polarization of the polarizers). As is well-known, from (62) it is possible to derive the Bell inequalities which are, therefore, necessary conditions for local realism. Many of these inequalities have been reported to be violated in experiments, but in all of them there exist loopholes for the refutation of LHV models [21,22]. Actually the inequalities which have been violated in experiments are derived from (62) *plus additional hypotheses which cannot be tested*. Typical of these is "no-enhancement" [6]. We shall see in the following that the additional hypotheses are naturally violated in a LHV model derived from the Wigner-function formalism of PDC.

In the analysis of the experiments, which we have made using the Wigner-function formalism [8–11], the joint detection probability appears in the form

$$P_{12} = const \times \int \mathrm{d}t_1 \int \mathrm{d}t_2 \langle (I_1(\mathbf{r}_1, t_1) - I_{01}) \times (I_2(\mathbf{r}_1, t_2) - I_{02}) \rangle, \quad (63)$$

where $I_1(I_2)$ is the light intensity arriving at the first (second) detector, placed at $\mathbf{r}_1(\mathbf{r}_2)$, at time $t_1$ ($t_2$). The integrals extend over appropriate detection time-windows, and $I_{01}, I_{02}$ are parameters corresponding to the average intensity of the zeropoint. Actually those modes of the radiation which contain only zeropoint, without additional radiation, contribute equally to the averages of $I_1$ and $I_{01}$, and therefore cancel out in the difference, and similarly for $I_2 - I_{02}$. Consequently only a few modes are needed in practice, but there is no problem if we include more modes than those strictly needed. In any case equation (63) was derived including only modes corresponding to a beam of almost parallel wave vectors. If this is not the case we should write equation (63) using the Poynting vector rather than the intensity (see below). The proportionality constant in equation (63) is irrelevant for many purposes, including the test of those "Bell inequalities" which are derived using additional hypotheses. But that constant is very relevant for the test of genuine Bell inequalities, derived from local realism alone [23]. Consequently we shall write equation (63) in a more complete form, which also shows the precise meaning of the average represented by $\langle\ \rangle$. We have

$$P_{12} = \int W(\{\alpha_{\mathbf{k}}\}, \{\alpha^*_{\mathbf{k}}\}) Q_1(\{\alpha_{\mathbf{k}}\}, \{\alpha^*_{\mathbf{k}}\}, \phi_1) \times Q_2(\{\alpha_{\mathbf{k}}\}, \{\alpha^*_{\mathbf{k}}\}, \phi_2) \mathrm{d}^N \alpha_{\mathbf{k}} \mathrm{d}^N \alpha^*_{\mathbf{k}}\,, \quad (64)$$

$$Q_j(\{\alpha_{\mathbf{k}}\}, \{\alpha^*_{\mathbf{k}}\}, \phi_j) = \eta_j (h\nu_j)^{-1} \times \int \mathrm{d}t_1 \int \mathrm{d}^2 r_j \left[ I_j(\{\alpha_{\mathbf{k}}\}, \{\alpha^*_{\mathbf{k}}\}, \phi_j, \mathbf{r}_j, t_j) - I_{0j} \right], \quad (65)$$

where $j = 1, 2$, $W$ is the "vacuum" Wigner function (Eq. (9)), $N$ is the number of modes (in practice we should take the limit $N \to \infty$ at some appropriate moment), $\mathbf{k}$ labels the wave vector and polarization of one mode. $I_1$ and $I_2$ are complicated functions of the amplitudes $\{\alpha_{\mathbf{k}}\}$ and $\{\alpha^*_{\mathbf{k}}\}$ which take account of the evolution, including the effect of the nonlinear crystal and the various optical devices present in the experiment (we refer to our articles [8–11] for details). These devices may contain controllable parameters which we have labeled $\phi_1$ and $\phi_2$. In addition to the time integration we have included an integration over the surface aperture of the detector. We have divided by the typical energy of one "photon" so that $Q_j$ becomes dimensionless. Finally $\eta_j$ is the quantum efficiency of the detector.

The relevant question is whether (64) may be considered a particular case of (62). If the answer is affirmative (negative) the formalism provides (does not provide) an explicit LHV model for the experiment, which therefore is (is not) compatible with local realism. We see that (64) looks precisely like (62) with the amplitudes $\{\alpha_{\mathbf{k}}\}$ and $\{\alpha^*_{\mathbf{k}}\}$ playing the role of the hidden variables $\lambda$. Indeed, the Wigner function $W$, playing the role of $\rho(\lambda)$, is positive definite (see Eq. (9)) and normalized. The problem appears with the positivity of $Q_j$. (The requirement $Q_j \leq 1$, certainly holds for the low quantum efficiencies of the experiments). Now $Q_j$ may be negative because the *difference* $I_1 - I_{01}$ (or $I_2 - I_{02}$) *is not always positive.* The problem is not the huge value of the zeropoint energy (about $10^5$ W/cm$^2$ for the ZPF in the visible range), because the threshold intensity $I_0$ cancels precisely that intensity. The problem lies in the fluctuation of the intensity. For the weak light signals of the experiments $I_j$ may have fluctuations such that $I_j < I_{0j}$. The problem of the non-positivity of $I_j - I_{0j}$ is alleviated by the time and space integrations in (65). Indeed, the fluctuations of the intensity are strongly reduced by those integrations as the Heisenberg (uncertainty) relations show. But we can guarantee the positivity of $Q_j$ only in the limit of infinitely wide time-windows and infinitely large apertures, which is non-physical. Consequently we conclude that it is not possible to interpret directly the Wigner-function formalism as a LHV model for the PDC experiments.

In spite of the above conclusion, it is not without interest to study whether some modification of $Q_j$ might give a LHV theory compatible with the experiments, though not precisely with the quantum predictions (which are given by the unmodified Eq. (64)). The question of a modification of quantum theory has been rejected by most, because of the spectacular success of that theory. However, we are not proposing any modification in the foundations of the quantum theory, but rather a change in the description of that complicated macroscopic system which is a photon counter. Equation (65) is the quantum prediction (in the Wigner representation) for the behaviour of an *ideal detector*. Every experimentalist knows that a *real device* is quite different from an ideal one in many respects. So what we propose is a realist theory of detection, in place of the model of instantaneous collapse which has been used hitherto.

An objection to any attempt at interpreting equation (65), or any small modification of it, as a "classical" (LHV) model is that any classical detector should be sensitive to the total radiation intensity, $I_j$, rather than to the difference $I_j - I_{0j}$. We do not agree with that. In fact, it is natural to assume that the detection should depend on the total flux of energy crossing the aperture during the time-window and this flux is given by an appropriate integral of the Poynting vector, rather than the intensity. Therefore we should write, instead of equation (65) the following

$$Q_j(\{\alpha_{\mathbf{k}}\}, \{\alpha^*_{\mathbf{k}}\}, \phi_j) = \eta_j (h\nu_j)^{-1} \times \int \mathrm{d}t_1 \int \mathrm{d}^2 r_j\ S_j(\{\alpha_{\mathbf{k}}\}, \{\alpha^*_{\mathbf{k}}\}, \phi_j, \mathbf{r}_j, t_j), \quad (66)$$

where $S_j$ is the component of the Poynting vector perpendicular to the entrance area of the detector. Equation (66) is not equal to equation (65), but the average shown in (64) *is the same in both cases.* The reason is that the average of the Poynting vector of the ZPF alone is zero, and the average of the contribution of the signal is the same whether we use the component of the Poynting vector or the intensity. Therefore we have not modified the quantum prediction up to this point. Still equation (66) is not positive definite, but now it is not difficult to imagine that a modification might be possible by making it positive without departing too much from the *ideal* quantum prediction equation (62). A model of such a detector has been presented elsewhere (for other purposes) which suggests that it is possible [24].

Finally we comment on the no-enhancement assumption. In a beam splitter (semitransparent mirror or polarizer) we should take into account both the signal and the "vacuum" zeropoint fields, so that at the outgoing channels (1 and 2) of this device, the electromagnetic fields $E_1^{(+)}$ and $E_2^{(+)}$, are given in terms of the incoming signal,

$E_{\rm S}^{(+)}$, and zeropoint, $E_{\rm ZP}^{(+)}$, fields by

$$E_1^{(+)}(\mathbf{r},t) = TE_{\rm S}^{(+)}(\mathbf{r},t) + {\rm i}RE_{\rm ZP}^{(+)}(\mathbf{r},t),$$
$$E_2^{(+)}(\mathbf{r},t) = TE_{\rm ZP}^{(+)}(\mathbf{r},t) + {\rm i}RE_{\rm S}^{(+)}(\mathbf{r},t), \qquad (67)$$

where $T$ ($R$) is the transmission (reflection) coefficient. This fact is essential in order to preserve the commutation relations in the beam-splitter [25].

The "no-enhancement assumption" essentially means that the intensity of a signal outgoing from a polarizer can never be greater than the incoming signal, and it is considered plausible because the beam-splitter divides the intensity. Equation (67) clearly shows that this assumption is naturally violated in any theory where the zeropoint field is real. The intensity at the outgoing channel may be greater than the intensity at the incoming channel for some realizations of the fields, although it will be certainly smaller on the average. A local realist theory, obtained by modifying the Wigner representation of quantum optics, along the lines discussed above, will violate both the "no-enhancement" assumption and the Bell-type inequalities derived from it.

In conclusion, parametric down conversion is an experimental arena for testing some important conceptual features of quantum mechanics. The Maxwellian approach introduced here offers a new perspective on the meaning of these experiments. On the other hand PDC is frequently used as a way to implement the techniques of quantum information, and our treatment can help by giving a better understanding of the correlation properties of this kind of light. Special attention has been paid to the spectrum of PDC, because this is one of the main features of the phenomenon. Two limit situations, usually found in the laboratory, have been studied, namely radius of the pumping beam much greater or much smaller than the length of the crystal times the sine of the angle between the laser axis and the wave vector of the emitted light. We recover the usual expressions for the matching conditions.

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